Offshore Stone Columns - Equipment, Quality Control and Outlook for Future Applications

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ABSTRACT

Stone columns are increasingly used in offshore applications in order to improve the soil bearing capacity, reduce settlement of foundations and to mitigate the potential of liquefaction. This paper describes typical applications and state-of-the-art installation techniques. In addition, key construction aspects of offshore technology are discussed, focusing on different installation methods and technical factors to be considered when selecting the equipment.

KEY WORDS: Stone Column; offshore; foundation; equipment; quality control.

INTRODUCTION

Stone columns are granular columns made of gravel size aggregate. The oldest method to install such columns onshore is known as Wet Top Feed. It is carried out by penetrating in to the ground a depth vibrator, or so called Vibroflot, and then feeding the gravel against the upstreaming flushing water down the hole that was previously washed out by the same Vibroflot, as shown in Figure 1. The column is built from the bottom to the top. The Vibroflot moves up/down and the number of such up/down stokes in each depth increment controls the produced diameter. In the marine environment, a 3.0 to 3.5 m thick gravel blanket is initially placed on the seabed. This blanket will feed the stone columns. The maximum stone column length that can be constructed using this method is in the order of 10 to 15 m as longer columns may be starved out of stone in the top meters of the columns.



Figure 1. Wet Top Feed installation

The stability of the annular space around the Vibroflot is achieved by maintaining the water level in the hole higher that the water table in the surrounding soil. However, this is not feasible when the installation is carried out under water, as shown in Figure 2. The lack of differential water head makes this simple installation method often not suitable for offshore applications (Hamidi et al., 2013).



Figure 2. Differential water head for onshore and offshore installation, , adapted from Al-Homoud and Degen (2006)

Therefore, the offshore installation of stone columns should use the Bottom Feed system, whereby the gravel is transported in a separate tremie pipe that is mounted alongside the Vibroflot (see Figure 3). In order to achieve a positive gravel flow (out of the tremie pipe and never in reverse direction), the tremie pipe needs to be put under an inside pressure that it at all times higher than the ambient pressure in the subsoil around the tip of the Vibroflot. Special equipment has been invented and patented (EP1367180A1) by the first author and applied on several sites to assure this controlled gravel flow at large water depths. This paper will mainly focus on the technology of this installation method.



Figure 3. Bottom Feed installation

OFFSHORE STONE COLUMNS PROJECTS

Seawall for Harbour of Patras, Greece

The marine structures of the old Patras Harbor in Greece were built directly on the seabed without ground improvement. The seabed consists of a normally consolidated soft clay layer that was 30 to 38 m thick. These structures were subject to settlements in the order of 3 to 4 m due to a series of moderate earthquakes of magnitude 3.5 to 4.5 that occurred in the Patras Gulf in 1984. Therefore, the breakwaters of the new harbor built around 2001 were founded on stone columns to increase the stability is seismic conditions. A typical section of the breakwater is shown in Figure 4. The stone columns were installed up to 20 m depth under up to 32 m water depth using bottom feed method. They were 1.0 m diameter and were arranged in triangular grids of 2.7 m to 3.3 m.



Figure 4. Typical breakwater section in the Harbour of Patras, adapted from Debats and Degen (2001)

Richards Bay, South Africa

In Richards Bay, South Africa a coal terminal was extended around 2002 with quay walls constructed with large concrete caissons. The natural seabed consisting of soft clay and silt was partially dredged and replaced with silty sand. Stone columns were adopted to both reduce the magnitude of the consolidation settlement of the natural seabed left in place and to increase the densification level of the reclaimed soil. Due to the variability of the soil to be treated, the stone columns were installed with variable diameter increasing the diameter in the layers that needed higher improvement to fulfill the design requirements. The installation was carried out up to 15m depth with bottom feed method from a barge prior to the placement of the caissons, as shown in Figure 5.



Figure 5. Stone columns installation at Richard Bay

Hong Kong Boundary Crossing Facility (HKBCF), Hong Kong

The HFBCF was constructed as part of the Zhuhai-Macau-Hong Kong Bridge project on 130 hectares reclaimed island. Due to environmental consideration, the reclamation was fully non-dredged for the first time ever in Hong Kong. Therefore, the marine deposits consisting of very soft clay and silts were left in place underneath the footprint of the entire island. The seawall consisted of 30 meters diameter cellular steel caissons with a rubble mound slope placed as wave protection on the sea side. A typical section of the seawall is shown in Figure 6. On this project over 1 million meters of stone columns were installed to both accelerate the consolidation of the soft sediments and to provide short term stability of the seawall during construction. The stone columns were in part installed offshore to a depth of over 38 m with an average diameter of 1.0 meter and in part they were installed onshore inside the previously constructed caissons.

One of the main challenges in this project consisted in the installation of offshore columns under severe height restrictions due to the vicinity to the existing Hong Kong International Airport. The bottom feed equipment provided by Betterground was specifically designed and

manufactured for this project in order to fulfill such requirements.



Figure 6. HKBCF typical seawall section (Arup Website, 2019)

STONE COLUMNS INSTALLATION EQUIPMENT

The Vibroflot

The depth vibrator known as vibroflot is the key equipment in the installation of stone columns. It is a machine that vibrates at a frequency of 30 Hz with amplitudes of typically over 20 mm. The vibration is created by a 130 kW rated electric motor that can for short peak times draw about double this energy. The vibration waves are horizontally polarized due to the motion of the vibroflot, which is beneficial to maximize the horizontal confinement stresses between soil and column.

A vibroflot has to be slim enough to penetrate through medium dense to dense soil while, on the other hand, should be large enough to contain a powerful electric motor and heavy enough eccentrical rotating weights to generate the vibration. Several vibroflot models have been developed in the past years, from those suitable to build small columns without inducing excessive vibrations in neighboring buildings to giant large area vibrators for compacting the sand of land reclamations. The B27 model described in Table 1 and Figure 7 has shown to be an ideally balanced design for offshore stone columns. It has excellent penetration capability and it well compacts sandy soils while installing stone columns.

Table 1. Features of vibroflot B27

Frequency	60 Hz	Eccentric Force	240 kN
Rotation Speed	1,800 rpm	Amplitude	24 mm
Motor Force	130 kW	Weight	2,200 kg



Figure 7. Vibroflot B27

Figure 8 below shows a B27 on a rig just hauled in for servicing. This type of equipment was operated from a 30 m x 30 m moon pool on a 4,000 ton jack up barge that can operate three such rigs at the same time. In this figure it is possible to observe the vibroflot nose cone (in the center) with an 8-inch diameter gravel tremie pipe (on the left) and a water jet pipe (on the right). Such jet pipe can be used to supply jetting water up to 12 bar pressure in order to aid the penetration of the vibroflot through hard or sticky soil layers.



Figure 8. Vibroflot with nose cone, tremie pipe and water jet pipe

The Double Lock System

Traditionally, the tremie pipe of the bottom feed system was maintained under pressure by injecting pressurized air and, at the same time, locking the top of the pipe with a single lock gate. However, this lock gate had to be periodically opened to feed a batch the stones into the tremie pipe. During such feeding operation, the tip of the vibroflot was rested into the previously constructed column in order to prevent the soil to flow into the pipe and clog the system (see Figure 9). During the years, the depth of stone columns on land-based sites was rising and, at some point, it became clear that for depths over 20 m there were problems with feeding the gravel out of the tremie pipe due to clogging. These issues occurred particularly in soft silty or clayey soil and under large ground water pressure. In such conditions, it became obvious that the shear strength of the column surrounding the tip of the vibroflot was not sufficient to resist the tendency of the soil to flow into the tremie pipe due to the differential pressure created when the single lock gate opened at the top of the rig. In order to overcome this problem, the so called "Double Lock" system was invented, which since then has safely prevented such "reverse circulation" of soil into the tremie pipe. This system should always be used for the installation of offshore stone columns in soft soil.



Figure 9. Stones feeding with single lock system

The functionality of the Double Lock system in offshore applications can be summarized as follows (refer to Figure 10):

- A stone column (19) has being installed by the vibroflot (15). During this operation, the lock tank (12) contains a batch of gravel and both lock gates are closed;
- The gravel is transported to the receiver tank (10) by either of these methods:
 - *a)* gravel flows via a so-called Gravel Pump from a hopper (5) into a blow tank (6) from where it is pumped by either water or air through a hose (8) to the receiver tank, or
 - *b)* gravel is transported to the receiver tank by a bucket (17) that runs on a second crane line;
- Gravel is stored in the receiver tank until the silo tube (14) is empty;
- The lower lock gate (13) opens and a full batch of gravel flows from the lock tank (12) into the silo tube (14). During this time the upper lock gate (11) must be maintained closed, to assure that the tremie pipe (16) is permanently under positive air;
- Once the lock tank is emptied into the silo tube, the lower lock gate closes and only thereafter the upper lock gate opens to let the gravel fall from the receiver tank to the lock tank.



Figure 10. Double Lock System, adapted from Degen (2014)

EQUIPMENT VARIATIONS AND THEIR SELECTION

General Considerations

Penetration and compaction capability of the vibroflot are generally the factors that influence the most the performance of the stone columns installation. However, the time efficiency of the system adopted to feed the gravel is also of great importance. In order to select the optimal gravel transport system in offshore application, the following parameters should be considered:

- Water depth (from highest water level to seabed);
- Column length (from seabed to maximum treatment depth); and
- Average column diameter, as it governs the total amount of gravel needed per each column.

The selection and set up of the equipment should also consider the waves and the water current for which the system has to be operational. When the installation is carried out in open water, the barge should have specific requirements to allow the functionality of the vibro equipment.

This section shows different set up solutions based on these general considerations.

Equipment for Shallow Water

In relatively shallow water and, in general, when the column length is much larger than the water depth, a skip bucket rig is a highly suitable feeding system. The skip bucket is filled on the barge and then travels up to the top of the rig to feed the stones through a hopper into the receiving tank. The skip bucket can travel independently from the up/down motion of the vibro equipment and the number of bucket supply cycles for each column is unlimited for this feeding process. Therefore, such system is useful to construct large diameter columns that require a large amount of gravel for each installation point.

There are several possible arrangements of the loading facilities on the deck of barge for the skip bucket system. Based on our experience, two types of arrangement are particularly efficient, as shown in the figures below. Figure 11 shows a skip bucket rig with a loading facility that can travel on a rail along the long side of the barge to meet the vibro rig at convenient locations, depending on the boom angle of the crane. The following Figure 12 shows a variant of the skip bucket loading system with the gravel supplied from a secondary barge. This has the great benefit that a standard land-based skip bucket rig can be used and the only specialty equipment required is the special gravel supply excavator with a trap door shovel.



Figure 11. Feeding system with skip bucket



Figure 12. Variant with skip bucket rig and secondary barge

The limitation of the skip bucket is that such system is not easily operated under water and, therefore, the top of the rig has to "stick out of the water" at any time during the installation to allow the gravel to flow from the skip bucket into the hopper. Hence, the bottom feed rig has to be built with a total minimum length equal to the treatment depth plus the water depth. For this reason, the skip bucket is generally adopted in shallow water where the cost of the extra length of the bottom feed rig is compensated by the simplicity and cost-effectiveness of feeding system.

Equipment for Deep Water

For water depths larger than approximately 10 m, it becomes desirable to have a vibro rig submersible. Generally, the rig is designed to have the top part "sticking out of the water" at least when the tip of the vibro is touching the seabed. A GPS antenna is fixed to the top of the rig. This allows to rely on the above-water D-GPS technology for navigation to set the coordinated of each compaction point. Adopting such design, the bottom feed rig has to be built with a total minimum length equal to the maximum between the treatment depth and the water depth. An example of equipment for deep water is shown below in Figure 13.



Figure 13. Submersible rig with water pump

When a submersible rig is used, the skip bucket is not suitable as a feeding method and the gravel has to be supplied via a Gravel Pump. In this situation, the gravel is transported by pressurized air or water and it is good practice to wash the stones before pumping them to reduce the risk of clogging the hoses. Figure 14 shows a vibro rig suspended from a crane beam at the beginning of a column installation (the top is still outside the water). In this set up, the vibro rig travels into position using a gantry crane. In this picture, it is possible to see two conveyor belts transported mixed with water through a 6-inch hose to the submersible vibro rig. Any skip bucket system would have been challenged to work in such geometry and at comparable speed.



Figure 14. Submersible rig with water pump

Equipment for Moderate Treatment Depth and/or Diameter

For a moderate column length and/or for small diameter columns, the loading of the vibro rig can be organized to happen only once per each stone column. This can generally be done when the total amount of gravel necessary to construct one column is less than 5 m^3 approximately, depending on the actual characteristics of the vibro equipment. An example of this arrangement is shown in Figure 15. The equipment shown in the sketch includes a lock tank and a hopper designed to have a capacity of 4.2 m^3 and 2.0 m^3 , respectively. In this example, the 6.2 m^3 of gravel that can be stored in the rig would allow to produce columns of 80 cm diameter up to approximately 12 m depth.



Figure 15. Equipment for single feeding cycle

Equipment to Work in Open Water

The installation of stone columns in open water should preferably be carried out by a jack up barge. Figure 16 shows a custom built 4000-ton platform that was used to install stone columns under the footprint of over 2 km of seawall in a Mediterranean port during almost the full year. The ability of this jack up platform to stand 5 m above the water table while operating with large gantry cranes limited significantly the interruptions due to waves and wind.



Figure 16. Jack up platform to work in open water

Figure 17 shows the three double lock rigs that were set up on the jack up platform. Two are suspended on cantilever beams on the left and the right of the barge, while the third rig operates in the moon pool. These rigs could be operational with wave height up to about 1.6 m.



Figure 17. Equipment set up on the jack up platform

The combined width of both sides of the platform is equal to the width of the work area inside the moon pool. This geometry allows to move the platform by the width of the moon pool and then "fill the gaps" in the stone column grids, as shown below in Figure 18. However, this also means that all three vibro rigs have to work at almost all times in coordination such that the respective work areas are completed at about the same time to achieve maximum efficiency. Despite this challenge, this project, that lasted for over two years, was completed ahead of schedule.



Figure 18. Underwater view of the jack up platform

The setup of the equipment in open water conditions should also consider the water current intensity. In 2006 the Bay Area Rapid Transit Authority (BART) commissioned a pilot trial for the retrofit of the subway tunnel that connects San Francisco with Oakland. During such trial, eightyeight stone columns were installed offshore as close as 1.5 m to the existing BART tunnel structure. The vibro equipment was a free hanging rig for deep water conditions operated without a vertical leads system. However, the water current in the Bay of San Francisco had an effect on the out-of-vertical attitude of the vibro rig stronger than expected. Due to the relatively small number of columns to be installed, the daily production time was reduced to maximum 30 minutes before and after the minimum tide, when the flow velocities of the water were low enough to allow the rig to penetrate sufficiently vertical into the soil.

The currents in the Bay of San Francisco were of much higher magnitude compared to any of the other projects that have been done with free hanging equipment, such as Patras, Richards Bay and other ports in the Mediterranean Sea. In future projects with similar strong currents, it is recommended that either piling leads or templates at the seabed are used to improve the verticality of the rig.

OFFSHORE QUALITY CONTROL

Monitoring and Operator Guidance System

Offshore stone columns are for their nature difficult to inspect. It is therefore crucial that an integrated monitoring and operator guidance system is adopted during installation to both achieve a high-quality product and provide quality control data to be reviewed post-installation. This system can be programmed with the relevant stone column installation parameters (treatment depth, column diameter as function of depth and soil type) in advance. The operator operates the crane wire rope driving the rig up and down to install the stone column following the real-time instructions provided by the system, as shown below in Figure 19. During such process, the gravel supply management, including the operation of the gravel pump, the multiple lock gates, and air and water supply valves are computer-controlled.



Figure 19. Monitoring and operator guidance system

Quality Control Data

Installation parameters are recorded in real time by the monitoring system. Quality control procedures based on the monitoring data can be successfully implemented only if such data is recorded at intervals of one second or less (Degen et al., 2017). One of the key monitored parameters is intensity of the electric current (amperage) necessary to vibrate the equipment into the soil. This parameter is well correlated to the type and density of the soil material encountered.

Figure 20 shows different ways to plot the installation parameters. These three plots consist of:

- Ampere over Depth on the left: this plot shows a strong increase in amperage at about 20 m depth, indicating the boundary between the softer soil that needs improvement and the underlying funding soil. In this graph it is not clear, however, if there are irregularities in the construction of the column.
- Ampere and Depth over Time in the middle: this plot shows a short penetration phase followed by a longer installation phase, during which the up/down motions of the vibroflot allow to build the column. It can be observed that around minute 8:00 the vibroflot was pulled out of the soil and then repenetrated to depth. In this plot it is noticeable the nature of the irregularity but it is not clear the effect on the diameter of the column.
- Column Diameter over Depth on the right: this plot refers to the same column shows in the middle plot. This profile is obtained by integrating the data of plot of the up-down motions with the information about the quantity of gravel supplied. In this plot a bottle neck in the column can be detected at the position of the black arrow.

All three above graphs are instructive for various aspects of quality control. This means that there is not one individual way to plot the quality control data for offshore stone columns and the contractor should provide all of them to get a full picture of the installation process.



Figure 20. Different ways to plot monitoring data

CURRENT APPLICATIONS OF OFFSHORE STONE COLUMNS

Offshore stone columns are currently used to improve the mechanical properties of the foundation soils underneath seawalls, breakwaters and reclamations. One of the main advantages of the stone columns is that the same product can be used to achieve different purposes, such as:

- provide immediate increase of the shear strength of natural soils to improve the slope stability during construction;
- provide a drainage path into cohesive soils to accelerate the consolidation process. Compared to prefabricated vertical drains, the drainage path offered by stone columns remains effective even after a significant settlement has already occurred;
- compact sandy soils to mitigate the potential of liquefaction.

Stone columns are a very ductile system that can overall cope well with the level of deformations often observed during the construction of marine works, especially when the founding soil consists of soft clayey and silty sediments. In addition, stone columns have the great advantage compared to piles and other ground improvement techniques that they can be installed with a variable diameter in order to concentrate the use of material and installation effort in the soil layers that require the highest level of improvement.

A typical example of the implementation of offshore variable diameter columns is the installation of tapered columns in normally consolidated marine deposits, as shown in Figure 21. In such soil conditions the undrained shear strength of the soil typically increases with depth. In addition, the upper layer of soil is subject to the largest shear stresses induced by the construction of the breakwater. Therefore, in this case it is desirable to install larger diameter columns in the top meters of the seabed.



Figure 21. Sketch adapted from CEDD Hong Kong (2003)

Another case suitable to the installation of variable diameter stone columns is the mitigation of liquefaction potential of subsoil that includes different material types. The quality control of stone columns installed under a shipping pier for such purpose is shown in Figure 22. In this project, the earthquake intensity was very high and the designer prescribed the installation of columns up to 1.60 m diameter in the silty sands and sandy silts. On the other hand, the underlying clean sand layer could be compacted upon vibration. The compaction effect makes the soil itself dilatant, hence not liquefiable, such that the reinforcement and drainage effect of a large column is less needed. Therefore, the column diameter could be reduced to 0.80 m in this layer. The high ampere reading observed for the sand in the quality control plot are indicative of a good compaction level achieved in this layer.



Figure 22. Variable diameter columns in layered soil

FUTURE APPLICATIONS FOR OFFSHORE STONE COLUMNS

Protection of Coastal Cities from Rising Sea Level

In the recent years, it became clear that the progressive rise of the mean sea level will demand the implementation of large projects to protect coastal cities from flooding. What was in the past a Dutch specialty, securing land that lies near the coast several meters below the sea level with a massive scheme of dikes and storm barriers, will be very soon exported around the world. Cities like Jakarta must execute bold plans to secure them from inundation in less than one or two decades from now. Figure 23 below is taken from an Indonesian newspaper and shows the size of such plans.

The experience gained with the artificial islands in Dubai can only be partially used here. In Dubai the water depth was 15 m maximum and the seabed was mostly composed of stable sandstone deposits. In many Asian coastal cities, the seabed near the shore consists of 10 m or more of marine deposits, comprising soft clays and silts. These soils cannot be treated by compaction like in Dubai, as clays do not compact under vibration like sands or gravels. The alternatives in such soil conditions are either cementitious ground improvement or stone columns. The latter method, if feasible in a project, is generally more cost effective. In addition, the use underwater of natural gravel instead of cement is preferable from an environmental point of view. Further considerations on design of seawalls founded on stone columns can be found in Degen et al. (2017).



Figure 23. Giant Sea Wall planning in Jakarta

Wind Turbine Foundations

Stone columns can be adopted to enhance the performance of wind turbine foundations and to provide more economical solutions. With reference to the different foundation systems shown in Figure 24, stone columns can be used with the following purposes:

- (a) Gravity-based foundations can benefit from stone columns to raise their bearing capacity and reduce total and differential settlement through a combination of soil reinforcement (silty and clayey soil) and compaction (sandy and gravely soil).
- (b) Monopile foundations can be more efficiently designed if stone columns increase the stiffness of the surrounding soil. The soil is laterally prestressed by the stone columns and can thereby provide more lateral support to the monopile. The stone columns also provide soil drainage to prevent the buildup of excess pore pressures under cyclic loading from wind and waves. Considering the improved lateral support given by the stone columns, smaller size monopiles should become feasible. Some monopile foundation suffers from very long-term lateral deformations that result in a progressive leaning of the pile. This can become relevant in the lifetime of the foundation as there are limits to the tolerated inclination. Stone columns installed with very high vibroflot ampere can cause in-situ soil deformations in the near range of such a leaning pile. Such induced deformation could counteract and, in some case, even reverse such unwanted rotation of a monopile in its upper sections.
- (c) Caisson foundations are less in number than gravity-based or monopile foundations, but they also can benefit from stone columns.
- (d) & (e) Stone columns installed with the double lock stone delivery system can be installed to water depths well over 30 m and can improve the bearing and lateral capacity of these foundation systems.



Figure 24. Sketch adapted from Kallehave (2015)

CONCLUSIONS

This paper compared the equipment used for the installation of stone columns onshore and offshore. In general, the main differences are:

- In offshore applications stone columns should always be installed with the bottom feed method, because the top feed method is not effective and reliable under water; and
- The use of the Double Lock system is recommended to prevented the "reverse circulation" of soil into the tremie pipe.

Different variations of the equipment are available for offshore stone columns. The skip bucket rig is usually preferable in shallow water while the feeding system with gravel pump is more suitable in deep water. Besides water depth, the amount of gravel needed per column is relevant for selecting the optimal equipment. If columns are either short enough or small enough in diameter to not consume more than a certain amount of gravel, a one-time filled rig instead of one with continuous gravel filling capability can in certain cases be more cost effective. It is therefore during the tender stage of projects important to have a clear

layout of the project geometries, informing about water depth, column length and average expected column diameter in all construction zones.

Real time monitoring data should be recorded at intervals of one second or less. The evaluation of this data should be undertaken by plotting:

- Ampere over Depth;
- Ampere and Depth over Time; and
- Column Diameter over Depth.

Stone columns are in the opinion of the authors not yet considered in a sufficient number of projects to complement or replace concrete or steel piling, for example in ways as suggested in Fig. 24 herein. This may be due to lack of understanding how columns can be produced offshore with sufficient quality and it is hoped that this paper helps to increase their acceptance among designers of offshore structures.

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